Mathematics

HSC Marking Feedback 2018

Question 11

Skills addressed:

- correctly multiply by $\frac{3-\sqrt{2}}{3-\sqrt{2}}$ (a)
- using the difference of 2 squares to expand the denominator (a)
- understanding the difference between a linear inequality and an absolute value inequality (b)
- understanding the need to reverse the inequality sign when dividing by a negative value (b)
- stating the two required equations and solving them simultaneously to find the common difference (di)
- showing the correct substitution into the formula for the $n^{th}$ term of an arithmetic series, evaluating $a$ and finding the 50th term (dii)
- finding the primitive and correctly evaluating the definite integral (e)
- listing expressions for $u, u', v$ and $v'$ and showing the correct substitution into the product rule (f)
- listing expressions for $u, u', v$ and $v'$ and showing the correct substitution into the quotient rule (g).

Areas for students to improve include:

- understanding why multiplying by a conjugate leads to a rational denominator (a)
- expanding binomial products (a)
- factorising before simplifying an algebraic fraction (c)
- using the Reference Sheet to obtain the rule for factorising the difference of 2 cubes (c)
- understanding that twentieth is $20^{th}$ not $12^{th}$ (di)
- using the Reference Sheet to obtain the formula for the $n^{th}$ term of an arithmetic series (di)
- understanding the difference between an arithmetic and a geometric series (di)
- selecting the appropriate formula to use in series questions (dii)
- understanding the rules for differentiating and integrating exponential functions (e)
- using the correct order when substituting limits, that is, $F(3) - F(0)$ instead of $F(0) - F(3)$ (e)
- evaluating $e^0$ correctly (e)
using the Reference Sheet to obtain the product rule and the rules for derivatives (f)
using the Reference Sheet to obtain the quotient rule and the rules for derivatives (g)
writing the function correctly when expressing the quotient as a product (g).

**Question 12**

Skills addressed:

- having a clear understanding and practical use of bearings and related notation (ai)
- presenting a correct equation noting the sum of two angles (ai)
- having a sound understanding of the use of both the sine and cosine rules (aii)
- quoting the correct formula with correct substitution (aii)
- ensuring final answer is provided as directed by the question (aii)
- copying the given diagram onto their answer sheet with correct labelling (ci)
- considering the linking of the various parts of a question (cii)
- carefully examining the question, considering if there is more than one approach and selecting the most appropriate and simplest method (cii)
- having a clear understanding of the difference between differentiation and integration and their use (di)
- having a clear understanding of the meaning of the term ‘stationary’ (dii).

Areas for students to improve include:

- being able to find the supplements and complements of given angles (ai)
- ensuring that they answer the given question as some students found the area of a triangle (ai)
- copying the correct formula from the Reference Sheet and then substituting the given values (aii)
- correctly following the order of operations in calculations (aii)
- completing all steps required as some students left their answer as $A C^2$ (aii)
- using the correct rule from the Reference Sheet to differentiate the trigonometric function (b)
- finding exact values of trigonometric expressions (b)
- providing a clear and sequential congruence proof with reasoning at each step (ci)
- using the properties of a square to equate sides (ci)
- correctly labelling pairs of angles and sides in their proof (ci)
- presenting correct and relevant reasoning for each step of their proof (ci)
- understanding and using congruence tests (ci)
- achieving the answer by subtracting twice the area of the triangle from the square (cii)
- correctly using the area formulae for each of the two relevant shapes (cii)
- understanding the definition and use of a perpendicular height (cii)
- linking the result found in (c i) to support the answer of (cii)
showing accuracy when substituting and evaluating expressions (di)
solving quadratic equations (dii)
ensuring that they fully answer the question as many students only found the time when the acceleration was zero (diii)
accurately substituting and evaluating expressions (diii).

Question 13

Skills addressed:
- showing numerical results in their testing of stationary points using either the first or second derivative (ai)
- using tables with labelled rows, indicating \( x \), \( \frac{dy}{dx} \) and slope, when using the first derivative test (ai)
- using a table of values to show the change in sign of the second derivative and hence a change in concavity (aii)
- indicating that they were substituting into the second derivative (aii)
- showing numerical results to verify their test (aii)
- stating a conclusion after their test (aii)
- factorising fully before solving \( \frac{dy}{dx} = 0 \) (ai)
- showing their numerical results when completing the testing of points (ai)
- stating conclusions by classifying their stationary points (ai)
- drawing a smooth curve of at least \( \frac{1}{3} \) of a page, clearly showing important features (aiii)
- labelling all important features, including intercepts (aiii)
- labelling the axes (aiii)
- drawing the diagram given in the question onto their answer sheet (bi)
- annotating their diagram with information provided in the question and geometric facts used in their proof (bi)
- using correct geometric reasons (bi)
- using an appropriate test for similarity and stated a conclusion (bi)
- clearly showing the pairs of equal matching angles (bi)
- drawing the triangles separately to help determine matching sides (bii)
- clearly showing the ratio of matching sides, for example, \( \frac{CB}{AB} = \frac{BD}{BC} \), before substituting the lengths given in the question (bii)
- correctly calculating the length of \( BD \) then \( AD \) (bii)
- showing the substitution of \( P(t) = 184 \) and \( t = 50 \) into \( P(t) = 92e^{kt} \) (ci)
- using logarithms to solve for \( k \) (ci)
- understanding that the variable for population was given ‘in millions’ (ci)
- showing the substitution of \( k = 0.0139 \) and \( t = 110 \) into \( P(t) = 92e^{kt} \) (cii)
Areas for students to improve include:

- remembering that the second derivative is used to test concavity not slope (aii)
- realising that the $y$-coordinate is given in the question, so there is no need to show it (aii)
- understanding the difference between a point of inflexion and a horizontal point of inflexion (aiii)
- sketching the curve beyond the domain of the features to be shown, including passing through the $x$-intercept of 6 (aiii)
- understanding the different tests for similar triangles (bi)
- stating matching angle pairs correctly, for example, $\angle ABC = \angle CDB$ (bi)
- setting out a proof, providing statements, in a logical order stating reasons at each step (bi)
- defining variables used that are not given in the question (bii)
- understanding that if using the SAS test for similarity that the included angle is required (bii)
- recalculating the value for $k$ if their result does not match value given in question (ci)
- if using 184 000 000 also use 92 000 000 (ci)
- understanding that $t$ is the number of years after 1910, so 2020 is represented by $t = 110$ (cii).

**Question 14**

Skills addressed:

- showing the appropriate substitution into the area of a triangle formula,
  - that is, $A = \frac{1}{2} \times 3 \times 6 \times \sin 60^\circ$ (ai)
- substituting the exact value of $\sin 60^\circ$ into the formula before simplification (ai)
- presenting a simplified exact value as their final answer (ai)
- understanding that ‘hence’ implies linking parts (ai) and (aii)
- carefully examining a question, considering if there is more than one approach, and selecting the most appropriate and simplest method (aii)
- recognising that their result in (ai) is equal to the sum of the areas of the two smaller triangles (aii)
- stating that $V = \pi \int_{1}^{10} (y - 1)^{\frac{1}{2}} \, dy$ (b)
- integrating and substituting limits correctly and presenting all intermediary steps (b)
- obtaining $f'(x)$, the discriminant of the derived function and knowing to use $\Delta < 0$ (c)
- correctly expressing their solution to the quadratic inequality (c)
- recognising the need to find the sum of an arithmetic and a geometric sequence (dii)
- correctly substituting into series formulas (dii)
drawing a clearly labelled tree diagram (e)

• having a clear understanding of when to use addition or multiplication of fractions in probability (e)

• understanding the meaning of ‘at least one’ in probability questions (e)

• using \( P(\text{at least one pen faulty}) = 1 - P(\text{both pens not faulty}) \) (ei)

• clearly presenting and using a tree diagram to establish the two correct options (eii)

• understanding that selecting machine A or machine B requires multiplying options by \( \frac{1}{2} \) (eii).

Areas for students to improve include:

• assuming information not stated in the question without proof, for example, \( \angle LKN = 90^\circ \) (ai)

• expressing an answer in exact form (aii)

• understanding that rotating a region about the \( y \)-axis requires the formula
  \[ V = \pi \int_{a}^{b} x^2 \, dy \], where \( a \) and \( b \) are \( y \)-values (b)

• using algebraic skills to find an expression for \( x^2 \) in terms of \( y \) (b)

• integrating using the reverse chain rule (b)

• choosing and applying the correct limits (b)

• understanding of the relevance of the discriminant (c)

• understanding how to find the discriminant (c)

• solving quadratic inequalities (c)

• understanding the difference between \( T_n \) and \( S_n \) (di)

• understanding that ‘on each of the first 3 days’ implies three terms are needed (di)

• substituting values into a given formula prior to obtaining the final answer (di)

• recognising different types of sequences (dii)

• correctly obtaining the first term of either the arithmetic or geometric sequence (dii)

• using arithmetic and geometric series formulae (dii)

• accuracy of calculations when systematically adding all 20 terms (dii)

• being accurate in calculations (ei)

• setting up a probability tree diagram (ei)

• understanding when to reduce fractions if an event is repeated (eii)

• correctly using a probability tree diagram (eii).
Question 15

Skills addressed:

- showing the substitution of \( t = 0 \) into \( L(t) = 12 + 2\cos\left(\frac{2\pi t}{366}\right) \) (ai)
- using the fact that \(-1 \leq \cos x \leq 1\) to find the range of \( L(t) \) and hence the solution (aii)
- drawing a graph to find the minimum value (aii)
- understanding how to work with trigonometric functions in terms of \( \pi \) (aiii)
- equating the two parts of equal area using definite integrals, finding primitive functions and solving the resulting logarithmic equation (b)
- knowing that \( k > 0 \) and discarding \( k = -15 \) (b)
- knowing the required area could be represented by \( A = \int_a^b (f(x) - g(x)) \, dx \) where \( a \) and \( b \) are \( x \)-values (ci)
- simplifying \( f(x) - g(x) \) before finding the primitive function and calculating the definite integral (ci)
- using the correct formula for Simpson’s rule and understanding that three function values are required for one application of the rule (cii)
- using their simplified function from (ci) to calculate their function values (cii)
- using a table to show their three functions values (cii)
- equating the gradient function for the tangent at \( P \) and the gradient of the line \( y = 2x \) and solving for \( x \) (ciii)
- finding the \( y \)-coordinate by substitution (ciii)
- understanding that the triangle is not right angled (civ)
- using the formula for perpendicular distance provided in the Reference Sheet (civ).

Areas for students to improve include:

- knowing that \( \cos 0 = 1 \) (ai)
- using the mark allocation as a guide to the amount of working expected (aii)
- understanding how to find ‘angles of any magnitude’ (aiii)
- changing the subject of the equation to \( t \) (aiii)
- understanding the difference between radians and degrees when solving trigonometric equations (aiii)
- using the correct order when substituting limits into the primitive function (b)
- solving logarithmic equations (b)
- writing correct statements, involving definite integrals, to use in area problems (ci)
- applying absolute value of functions correctly (ci)
- using the formula from the reference sheet and understanding the meaning of \( \frac{b-a}{6} \) (cii)
- finding the correct function to use in Simpson’s rule (cii)
- showing substitutions used to find the \( y \)-coordinate (ciii)
• performing calculations with surds (ciii)
• understanding that the equation of the line used in the perpendicular distance formula needs to be in general form (civ)
• finding the correct angle at the origin to use in the sine rule (civ).

Question 16

Skills addressed:

• having a clear understanding of the steps required to ‘show’ a result (ai)
• using Pythagoras’ theorem to correctly link the two given variables (ai)
• showing their substitution of information into the given volume formula (ai)
• listing expressions for \( u, u', v \) and \( v' \) and showing the correct substitution into the product rule (aii)
• simplifying their expression for \( \frac{dv}{dx} \) showing setting out in logical steps (aii)
• understanding the process used for finding a maximum or minimum value (aiii)
• ensuring that they demonstrate a test to distinguish between minimum and maximum results (aiii)
• selecting a method to ensure all possible elements in the event and sample space are counted systematically (bi)
• presenting a table which identifies all options for each possible combination (bi)
• using the counting system established in part (bi) to support the solution in (bii)
• grouping options and set work out in a methodical manner (bii)
• having a clear understanding of the steps required to ‘show’ a result (c)
• presenting all work clear sequential steps (c)
• providing a detailed progression from \( A_1 \) through to \( A_2 \). (ci)
• responding to the direction ‘show that’ and providing a detailed progression from \( A_2 \) to \( A_3 \) (cii)
• being able to achieve an expression for \( A_n \) (cii)
• using the sum of a geometric progression formula to arrive at the given result (ciii)
• demonstrating a high degree of accuracy and skill in algebraic manipulation (ciii).

Areas for students to improve include:

• writing a correct statement for Pythagoras’ theorem and rearranging the resulting equation (ai)
• using the pronumerals given in the question (ai)
• differentiating functions requiring the use of the chain rule (aii)
• manipulating algebraic fractions and leaving the result in simplified form (aii)
• omitting solutions for \( \frac{dv}{dx} = 0 \) which are invalid, that is, \( x = 0 \) (aiii)
• showing numerical results when completing the test for the maximum or minimum
value (aiii)

- using their $x$-value to find $\theta$ as required (aiii)
- developing a range of counting techniques to address the variety of stimulus found in probability questions (bi)
- understanding to have ‘no chance of winning before rolling the third die’, a double or consecutive number needed to be thrown (bi)
- recognising all possibilities available to achieve a win (bii)
- realising that the idea of a complement was not required (bii)
- using brackets correctly (ci)
- remembering to increase and subtract the withdrawal, that is, using $A_2 = A_1(1.04) - P(1.05)$ (ci)
- knowing that the third withdrawal was $P(1.05)^2$ and using $A_3 = A_2(1.04) - P(1.05)^2$ (cii)
- using patterns to obtain an expression for $A_n$ (ciii)
- using the correct values for the first term and common ratio when finding the sum of the geometric progression (ciii).